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MODIFIED DUGDALE APPROACH TOCOHESIVE QUADRATIC LOAD DISTRIBUTION ARRESTING CRACK OPENING

Ravinder Kumar, Dr. Pradeep Goel

Associate Professor M. M. CollegeFatehabad, Haryana
Research Scholar, CMJ University, Shillong, Meghalaya

ABSTRACT

The Dugdale model for a single straight slit in an infinite plane has been extended to the case of two straight cracks in an infinite plane proposed by Theocaris (55). It is further modified by him using a stepwise approximation for obtaining the solution of two collinear straight cracks where the plastic zones developed are closed by variable load distribution over their rims.

INTRODUCTION

An infinite, homogeneous, isotropic, elastic-perfectly plastic infinite plate, bounded by xoy plane, is cutalong two hairline straight cracks L_1 and L_2. These equal and collinear cracks lie on ox-axis and aresymmetrically situated about oy-axis. The crack L_1 lies from (—b, 0) to (—a, 0) and L_2 lies from (a, 0) to (b, 0) unidirectional tension, σ_∞ is applied at infinite boundary in direction perpendicular to the rims of the cracks L_1 and L_2. Consequently the faces of the cracks open forming small plastic zones ahead of tips of the cracks. The plastic zones developed at the four tips –b, -a, a and b are denoted by Γ_4, Γ_3, Γ_2 and Γ_1 , respectively. The plastic zone Γ_1 occupies the region [b, d]; the interval [c, a] is occupied by plastic zone Γ_2; the plastic zone Γ_3 occupies [- a, -c] and Γ_4 occupies the interval [- d, -b].

Each rim of the plastic zones Γ_i (I = 1, 2, 3, 4) is subjected to the compressive stress distribution

$P_{yy} = t^2 \sigma_{ye}$ and $P_{xy} = 0$. Any point on the rim is denoted by t $_{ye}$ is yield point stress of the plate. and σ_{ye}

Thus the cracks are arrested from further opening.

The entire configuration is depicted in figure 4.1

CONCLUSION

The conclusion of the above stated problem is obtained using principle of super positions of stress intensity factors at the tips of the cracks obtained for two component problems. These problems are appropriately derived from problem stated in section 4.1. These problems are termed problem I and problem II. These are stated and solved below.

Material and method:

Problem I

An infinite, homogeneous, isotropic, elastic-perfectly plastic plate occupies xoy plane. The plate is cut along two straight cracks R_1: $\Gamma_4 UL_1 U\Gamma_3$ and R_2: $\Gamma_2\ UL_2 U\Gamma_1$. The cuts R_1 and R_2 occupy the interval [-d, -c] and [c, d] respectively.

The boundary conditions of problem are

(i) No stresses are acting on the rims of the cracks R_1 and R_2.

(ii) The stress prescribed at infinite boundary is $P_{yy} = \sigma_\infty$, $P_{xy} = 0$, $P_{xx} = 0$.

(iii) The displacements are single valued on and around the cracks.

This problem is the same as stated in section 3.2.1 and depicted in figure 3.2 of chapter 3. We recapitulatethe solution from 3.2.1 make this chapter self-sufficient.

The complex potential $\phi^I(z)$, of interest may directly be written as

$$\phi^I(z) = \frac{\sigma_\infty}{2}\left[\frac{1}{iX(z)}\left\{z^2 - \frac{d^2 E(k')}{K(k')}\right\} - \frac{1}{2}\right], \qquad (4.2.1\text{-}1)$$

Where $X(z) = \{(z^2 - c^2)(d^2 - z^2)\}^{1/2}$ (4.2.1-2)

And complementary modulus $k' = (1 - c^2/d^2)^{1/2}$

The opening mode stress intensity factor at interior tip z = c is

$$(K_{I_c}^{I}) = \frac{\sigma_\infty d^2}{i}\sqrt{\frac{\pi}{c(d^2 - c^2)}}\left[\frac{c^2}{d^2} - \frac{E(k')}{K(k')}\right] \qquad (4.2.1\text{-}3)$$

And at the exterior tip z = d as

$$(K_{I_d}^{II}) = \sigma_\infty d\sqrt{\frac{\pi d}{(d^2 - c^2)}}\left[1 - \frac{E(k')}{K(k')}\right] \qquad (4.2.1\text{-}4)$$

Problem II

A homogeneous, isotropic and elastic-perfectly plastic infinite plane is bounded by xoy plane. Two hairline cracks L_1 and L_2 lie on ox-axis of the infinite plate. The cracks L_1 and L_2 occupy the interval [-b, -a] and [a, b] respectively. Remotely applied unidirectional (parallel to oy axis) uniform tension, σ_∞, causes the opening of rims of the cracks. These are denoted by Γ_1, Γ_2, Γ_3 and Γ_4 and lie ahead of the tips b, a, -a and -b respectively. The interval occupied on the real axis by the plastic zone Γ_1 is [b, d]; by Γ_2 is [c, a]; by Γ_3 is [-a, -c] and by Γ_4 is [-d, -b].

Each rim of the plastic zones Γ_i (i = 1, 2, 3, 4) is subjected to tensile quadratically varying stress distribution $P_{yy} = t^2\sigma_{ye}$, $P_{xy} = 0$ and $p_{xx} = \sigma_{ye}$. 0 denotes the yield point stress and t is a point on any rim of any of the plastic zones.

The configuration of problem II is depicted in figure 4.2.

The mathematical model of the above problem is obtained assuming two cracks R_1 and R_2 effectively formed by $\Gamma_4 \cup L_1 \cup \Gamma_3$ (=R_1), $\Gamma_2 \cup L_2 \cup \Gamma_1$ (=R_2) and lying on the ox axis of the infinite plate. The boundary conditions of the problem may be restated as

(a) The cracks R_1 and R_2 are loaded along the rims of r_i (i = 1, 2, 3, 4)k by stress distribution
$P_{yy} = t^2\sigma_{ye}$, $P_{xy} = 0$, $P_{xy} = 0$

(b) The rims of L_1 and L_2 are stress free.

(c) No stresses are acting at infinite of the plate.

Using boundary conditions (a), (b) and equation (2.5-5) of chapter 2 following two Hilbert problems areobtained.

$$[\phi^{II}(t) + \Omega^{II}(t)]^{+} + [\phi^{II}(t) + \Omega^{II}(t)]^{-} = t^{2}\sigma_{ye} \quad (4.2.2\text{-}1)$$

$$[\phi^{II}(t) - \Omega^{II}(t)]^{+} + [\phi^{II}(t) - \Omega^{II}(t)]^{-} = 0, \quad (4.2.2\text{-}2)$$

Where $\Gamma = \bigcup_{i=1}^{4} \Gamma_{1}$ Superscript II denotes that the potentials refer to problem II.

The solution of equations (4.2.2-1) and (4.2.2.-2) may be written using equation (2.5-16) and (2.5-17) as

$$\phi^{II}(z) = \phi_{0}(z) + \frac{1}{X(z)}[C_{0}z^{2} + C_{1}z + C_{2}], \quad (4.2.2\text{-}3)$$

$$= \Omega^{II}(z),$$

Where

$$\phi_{0}(z) = \frac{\sigma_{ye}}{2\pi i X(z)} \int_{r} \frac{t^{2}X(t)}{(t - z)} dt. \quad (4.2.2\text{-}4)$$

And X(z) is same as defined by (4.2.1-2). The constants $C_i(I = 0, 1,2)$ are determined using condition (c) stated above and the condition of single valuendness of displacement around cracks. This gives

$$C_{0} = 0 \quad (4.2.2\text{-}5)$$

$$C_{1} = \frac{\sigma_{ye}}{\pi i}[-\frac{a^{2} + 2c^{2} + 2d^{2}}{3a}X(a) + \frac{b}{3}X(b) + \frac{c^{2} + d^{2}}{a}X(a) + \frac{c^{2} + d^{2}}{2a}X(a)] \quad (4.2.2\text{-}6)$$

And

$$C_{2} = 0 \quad (4.2.2\text{-}7)$$

Evaluating integral of the equation (4.2.2-4) complex potential $\phi_0(z)$ may be written as

$$\phi_0(z) = -\frac{z\sigma_{ye}}{\pi i X(z)}\Big[\frac{d}{3}\{2(c^2+d^2)H_o - c^2G\}_0 - \frac{a^2+2c^2+2d^2}{3a}X(a) + \frac{b}{3}X(b)$$

$$-(c^2+d^2-z^2)\{dH_0 - \frac{X(a)}{a}\} - \frac{X^2(z)}{d}G_1 + \frac{z^2X^2(z)}{d}P_0\Big],$$

(4.2.2-8)

Where

$$H_0 = E(u_1) + E(u_2), \tag{4.2.2-9}$$

$$G_0 = F(u_1) + F(u_2), \tag{4.2.2-10}$$

$$G_1 = u_1 + u_2 \tag{4.2.2-11}$$

$$P_0 = \Big[\frac{1}{z^2}\{u_1 + \frac{c^2}{(z^2-c^2)}\Pi(u_1, \alpha_1)\}^2 + \frac{1}{(d^2-z^2)}\Pi(u_2, \alpha_2)\Big]^2, \tag{4.2.2-12}$$

And $F(u_1)$, $F(u_2)$, $E(u_1)$, $E(u_2)$, Π (u_1, α_1^2)] and Π (u_2, α_2^2) are normal elliptic integrals of the first, second and kind respectively. Also

$$u_1 = \sin^{-1}\frac{d}{a}\sqrt{\frac{(a^2-c^2)}{(d^2-c^2)}},\ u_2 = \sin^{-1}\sqrt{\frac{(d^2-b^2)}{(d^2-c^2)}},$$

X(z) is same as defined by equation (4.2.1-2). These yield

The opening mode stress intensity factor (SIF), K_I^{II}, at the interior tip z = c is obtained substituting value of $\phi^{II}(z)$, from equation (4.2.2-3 to 12), for $\phi(z)$ in equation (2.6-1) one obtains.

$$(K_I^{II})_c = -\frac{2\sigma_{ye}}{i}\sqrt{\frac{c}{\pi(d^2-c^2)}}\Big[\frac{d}{3}\{2(c^2+d^2)H_0 - c^2G_0\} - d^3H_0 - \frac{(3c^2+d^2)}{2a}X(a)\Big],$$

(4.2.2-13)

And SIF exterior tip z = d may be written as

$$(K_I^{II})_d = -2\sigma_{ye}\sqrt{\frac{d}{\pi(d^2-c^2)}}\left[\frac{d}{3}\{2(c^2+d^2)H_0 - c^2G_0\} - c^2 dH_0 - \frac{(3d^2+c^2)}{2a}X(a)\right], \qquad (4.2.2\text{-}14)$$

CONCLUSION

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